Mechanical characterization

(Resistance to deformation and fracture)

Jesús Rodríguez

Universidad Rey Juan Carlos

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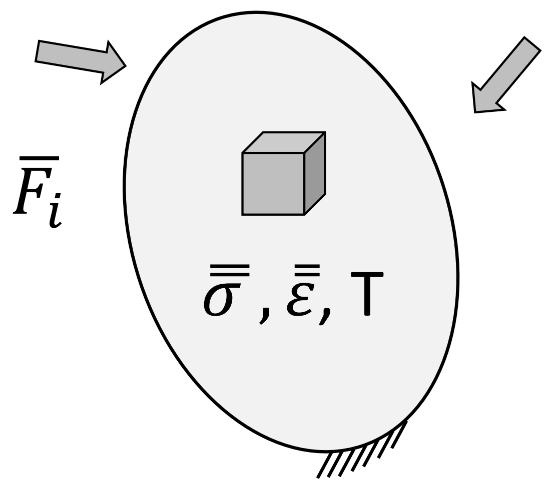
Objective

- The aim of this lesson is to differentiate from a qualitative point of view and to identify the most common types of mechanical material behaviours.



Outline

- Phenomenological models: fundamentals and main characteristics
- Experimental tests
- Schematic models of real behaviour
- Examples of mechanical behaviour in additively manufactured materials

Phenomenological models

General considerations

- Phenomenology is based on observed experimental results looking for a relationship between cause and effect, which is know as a constitutive law.
- The approach is based on intrinsic properties independent of body geometry and definable in a volume element representative of the material as a whole (continuum mechanics hypothesis).
- The measurable magnitudes of Mechanics are force, displacement, time and temperature. The rest of magnitudes are derived under certain hypotheses.

Phenomenological models

Approaches to formulate constitutive laws

- Microscopic approach describes the observable macroscopic behaviour from the atomic, molecular or crystal levels as a result of an integrating or averaging process up to the volume element.
- The thermodynamic of continuous media approach explains the material behaviour in terms of macroscopic internal state variables.
- The functional approach is based on hereditary laws of integral type, describing the history of internal variables.

Phenomenological models

Hypotheses of the global phenomenological method

- The method consist of describing the behaviour of the volume element through the relationships between the variables constituting the input and output of the process under study (stress, strain and their rates, temperature, pressure, time, etc).
- Thermodynamics gives the general formulation of models without specifying their analytical form which must be determined through experiments.

$$\Psi\left(\bar{\bar{\sigma}}, \bar{\bar{\varepsilon}}, \dot{\bar{\bar{\varepsilon}}}, T, p, etc\right) = 0$$

Experimental tests

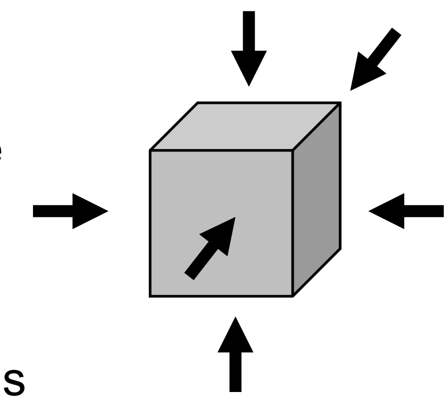
Objective $\Rightarrow$ $\Psi\left(\bar{\bar{\sigma}}, \bar{\bar{\varepsilon}}, \dot{\bar{\bar{\varepsilon}}}, T, p, etc\right) = 0$

$$\bar{\bar{\sigma}} = \begin{pmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{pmatrix}$$

$$\bar{\bar{\varepsilon}} = \begin{pmatrix} \varepsilon_x & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_y & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_z \end{pmatrix}$$

A general mechanical characterization requires **multiaxial tests**:

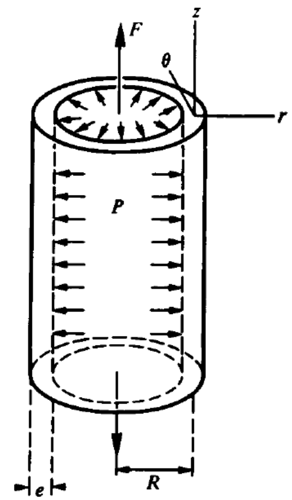
- Triaxial tension-compression
- Tubes under tension-compression and internal pressure
- Tubes under tension-compression and torsion
- Biaxial on cruciform specimens



Hypotheses: uniform deformation and isostatic conditions

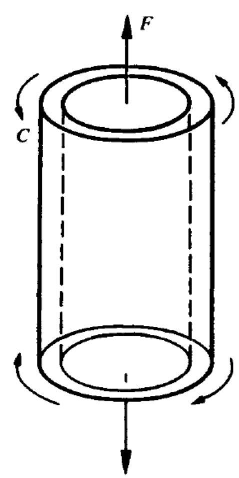
Experimental tests

- Tension-compression and internal pressure



$$\bar{\bar{\sigma}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & P\dfrac{R}{e} & 0 \\ 0 & 0 & P\dfrac{R}{e} + \dfrac{F}{S} \end{pmatrix}$$

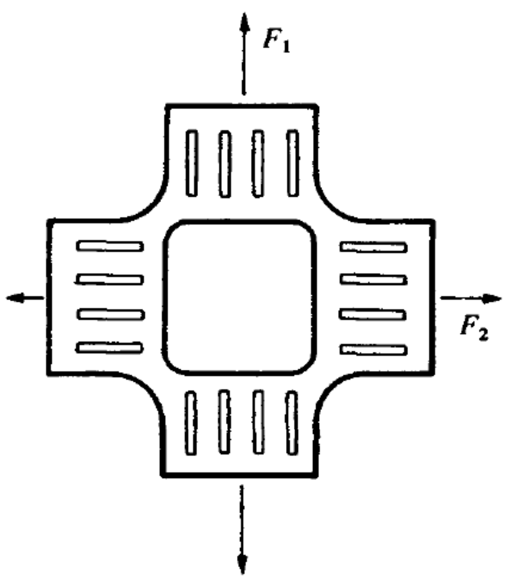
- Tension-compression and torsion



$$\bar{\bar{\sigma}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \dfrac{CR}{I} \\ 0 & \dfrac{CR}{I} & \dfrac{F}{S} \end{pmatrix}$$

Experimental tests

- Biaxial tests on cruciform specimens

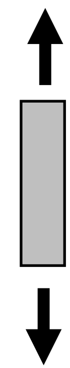


$$\bar{\bar{\sigma}} = \begin{pmatrix} \alpha F_1 - \beta F_2 & 0 & 0 \\ 0 & \alpha F_2 - \beta F_1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

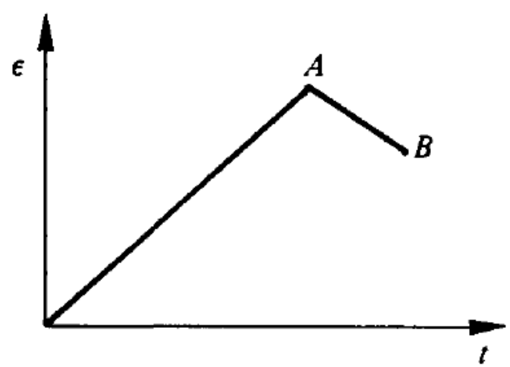
Experimental tests

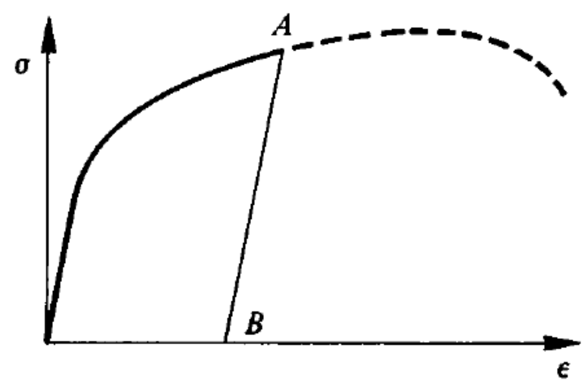
Uniaxial tests

- Hardening test in simple tension or compression



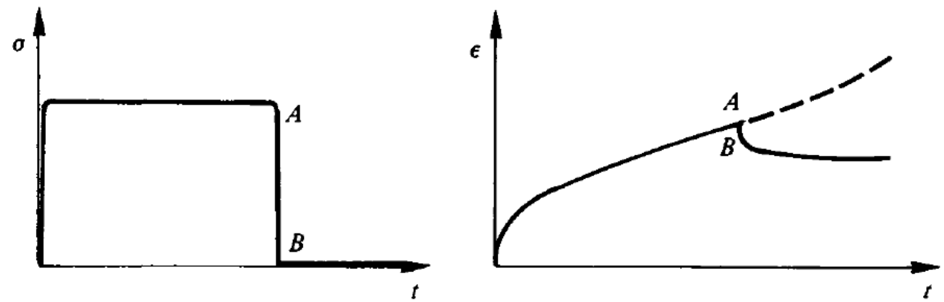
$$\bar{\bar{\sigma}} = \begin{pmatrix} \sigma & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \bar{\bar{\varepsilon}} = \begin{pmatrix} \varepsilon & 0 & 0 \\ 0 & \varepsilon_{\perp} & 0 \\ 0 & 0 & \varepsilon_{\perp} \end{pmatrix}$$

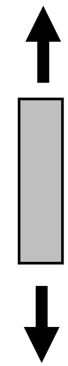




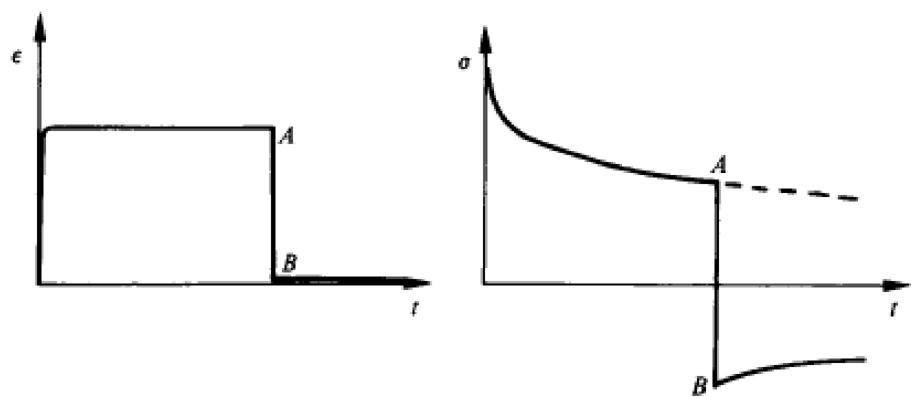
Experimental tests

- Creep test in simple tension or compression





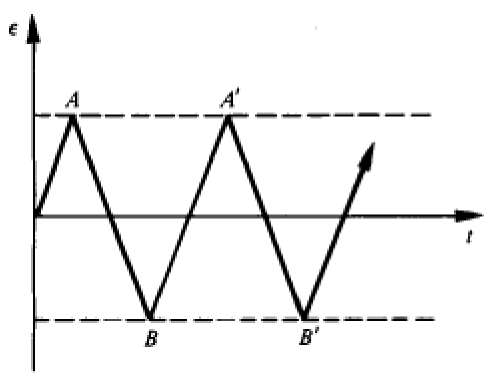
- Relaxation test in simple tension or compression

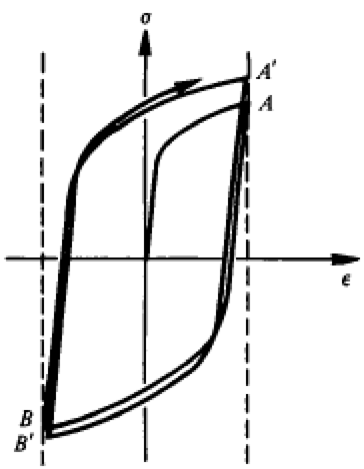


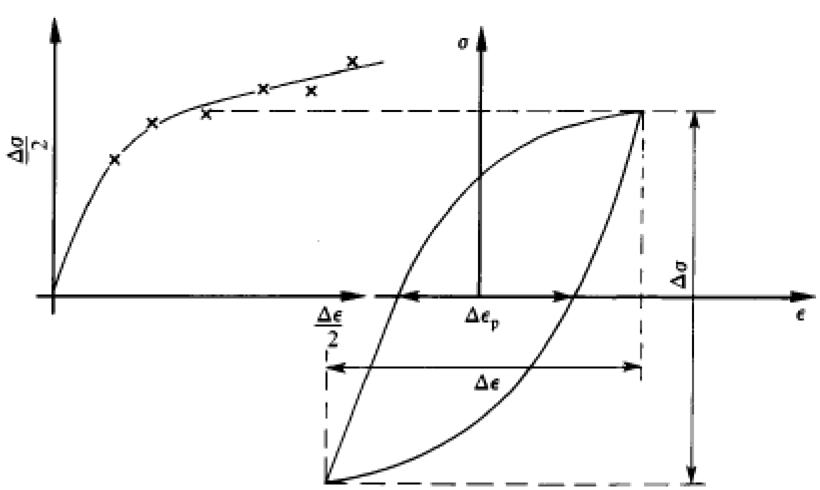
Experimental tests

- Cyclic test

The specimen is subjected to a periodic load (stress or strain) and the evolution of the cyclic response is studied.







Schematic representation of real behaviour

Models of mechanical behaviour of materials

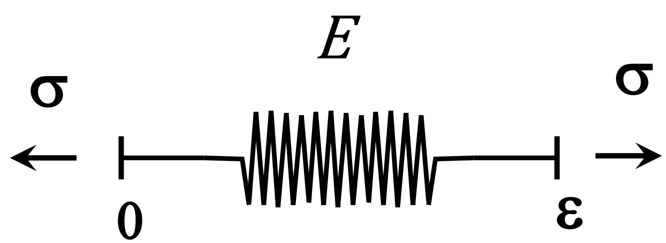
- Elastic (isotropic linear, isotropic nonlinear, anisotropic).
- Viscoelastic (linear or non linear)
- Plastic (rigid-perfectly plastic, elastoplastic with hardening).
- Viscoplastic (perfectly viscoplastic, elastoviscoplastic).
- Damageable (by deformation, by fatigue, etc).

Analogical models

- Assemblies of mechanical elements with responses similar to those expected in the real material.

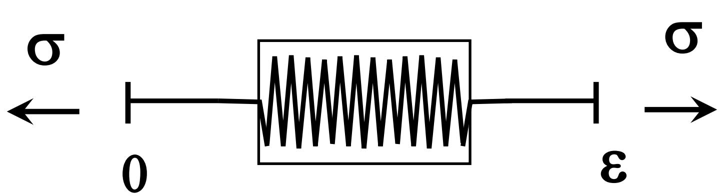
Mechanical elements

- Linear spring → Linear elasticity



$$\sigma = E\varepsilon$$

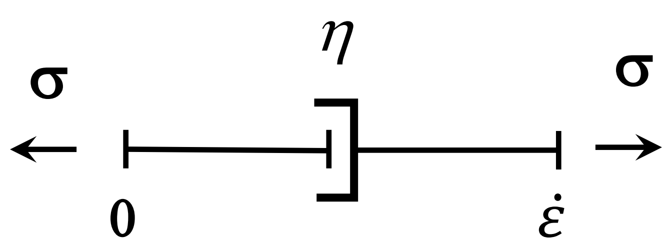
- Nonlinear spring → Nonlinear elasticity



$$\sigma = f(\varepsilon)$$

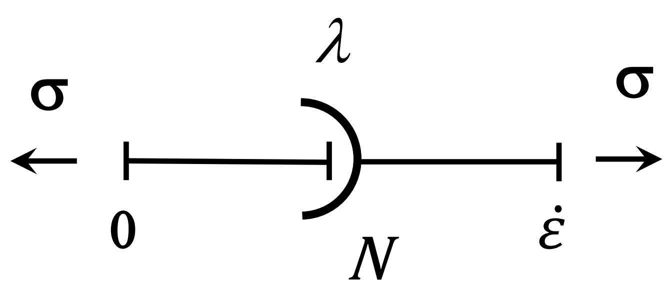
Mechanical elements

- Linear damper ⇒ Linear viscosity



$$\sigma = \eta\dot{\varepsilon}$$

- Nonlinear damper ⇒ Nonlinear viscosity

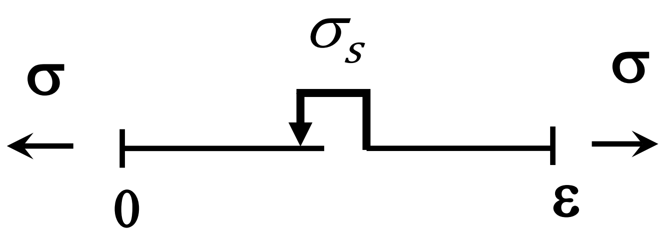


$$\sigma = f(\dot{\varepsilon}) \qquad \sigma = \lambda\dot{\varepsilon}^{\frac{1}{N}}$$

Mechanical elements

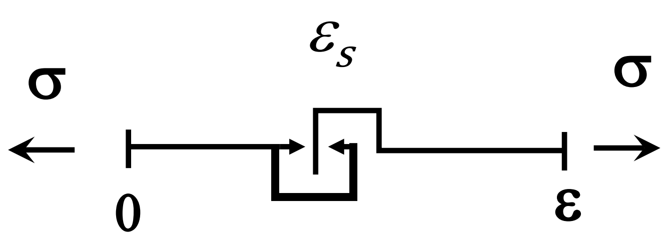
- Skidding block → Stress threshold

$(-\sigma_s < \sigma < \sigma_s)$



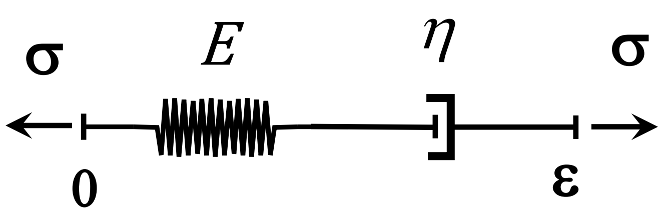
- Stopping block → Strain threshold

$(-\varepsilon_s \leq \varepsilon \leq \varepsilon_s)$



Mechanical elements assemblies

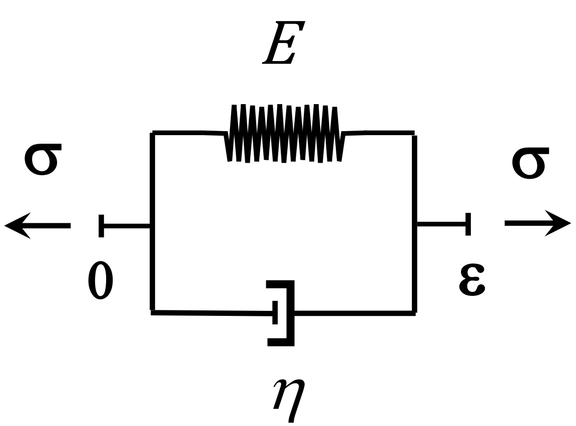
- In series



$$\sigma = \sigma_i$$

$$\varepsilon = \sum \varepsilon_i$$

- In parallel

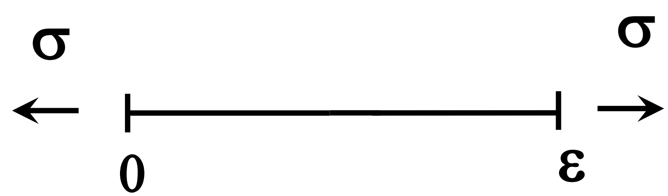


$$\varepsilon = \varepsilon_i$$

$$\sigma = \sum \sigma_i$$

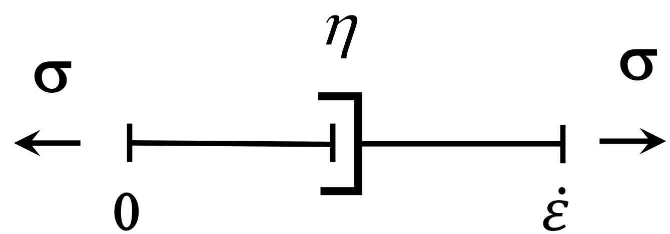
Material models

- Rigid solid



$$\varepsilon = 0, \forall\sigma$$

- Perfect fluid



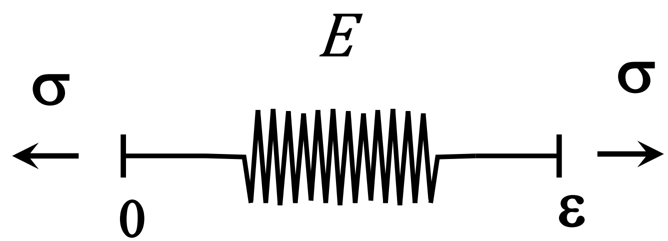
$$\sigma = \eta\dot{\varepsilon}$$

$$\varepsilon \rightarrow \infty, \forall\sigma$$

These models do not belong to the mechanical of deformable solids, but they may be interesting from the engineering point of view.

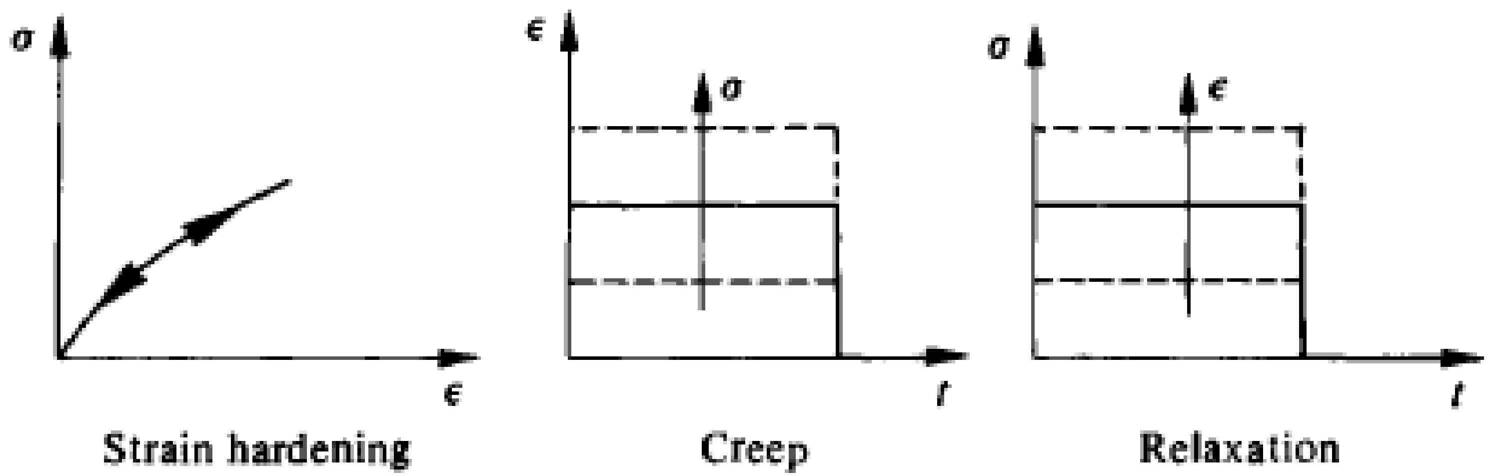
Material models

- Elastic solid



$$\sigma = E\varepsilon$$
$$\sigma = f(\varepsilon)$$

(Reversible, instantaneous and single-valued deformation)

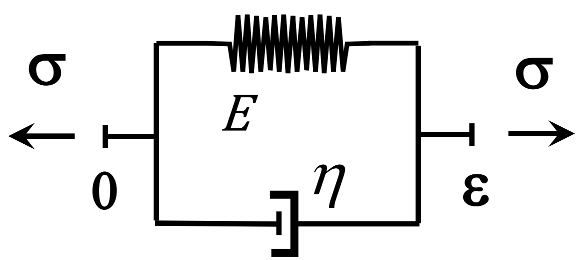


Material models

- Viscoelastic solid

Time dependent elasticity and delayed deformation.

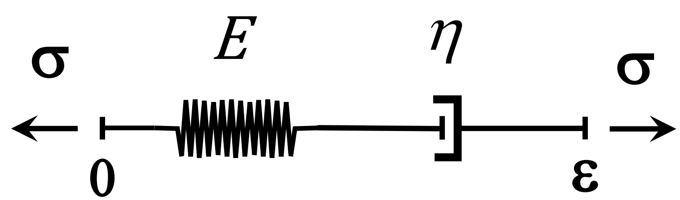
Kelvin-Voigt



$$\sigma = \eta\dot{\varepsilon} + E\varepsilon$$

$$E_{rel}(t) = E + \frac{\eta}{t}$$

Maxwell



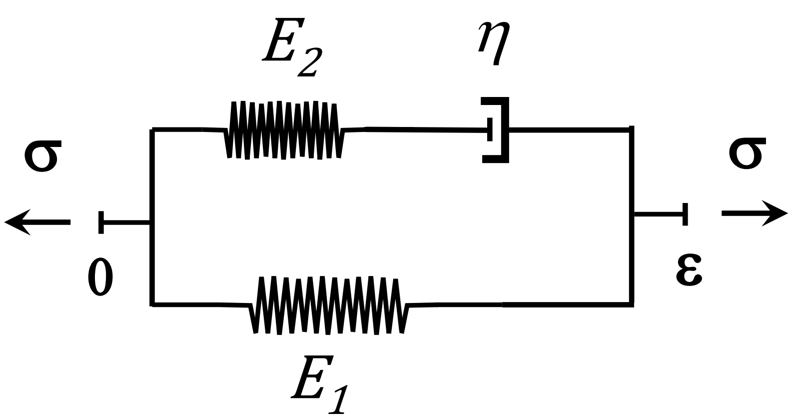
$$\eta\dot{\sigma} + E\sigma = \eta E\dot{\varepsilon}$$

$$E_{rel}(t) = E\ e^{-\frac{E}{\eta}t}$$

Neither of them meets the conditions required for a viscoelastic material

Material models

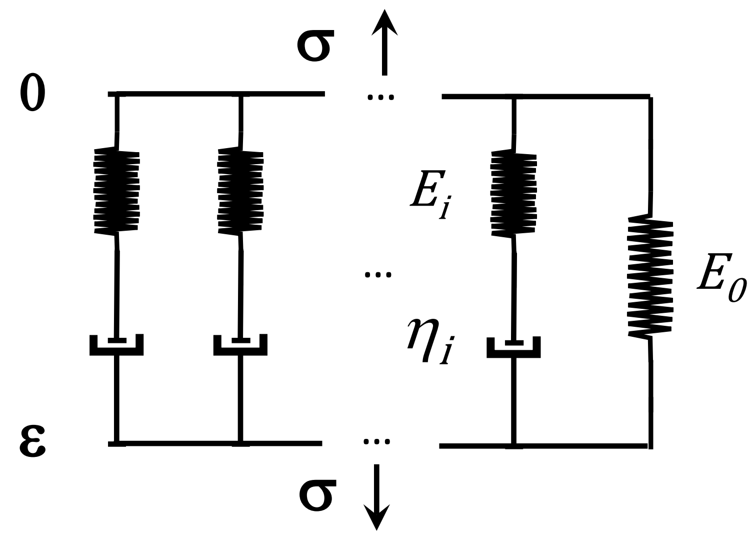
Viscoelastic standard solid



$$\eta\dot{\sigma} + E_2\sigma = \eta(E_1 + E_2)\dot{\varepsilon} + E_1 E_2 \varepsilon$$

$$E_{rel}(t) = E_1 + E_2\, e^{-\frac{E_2}{\eta}t}$$

Generalised viscoelastic model

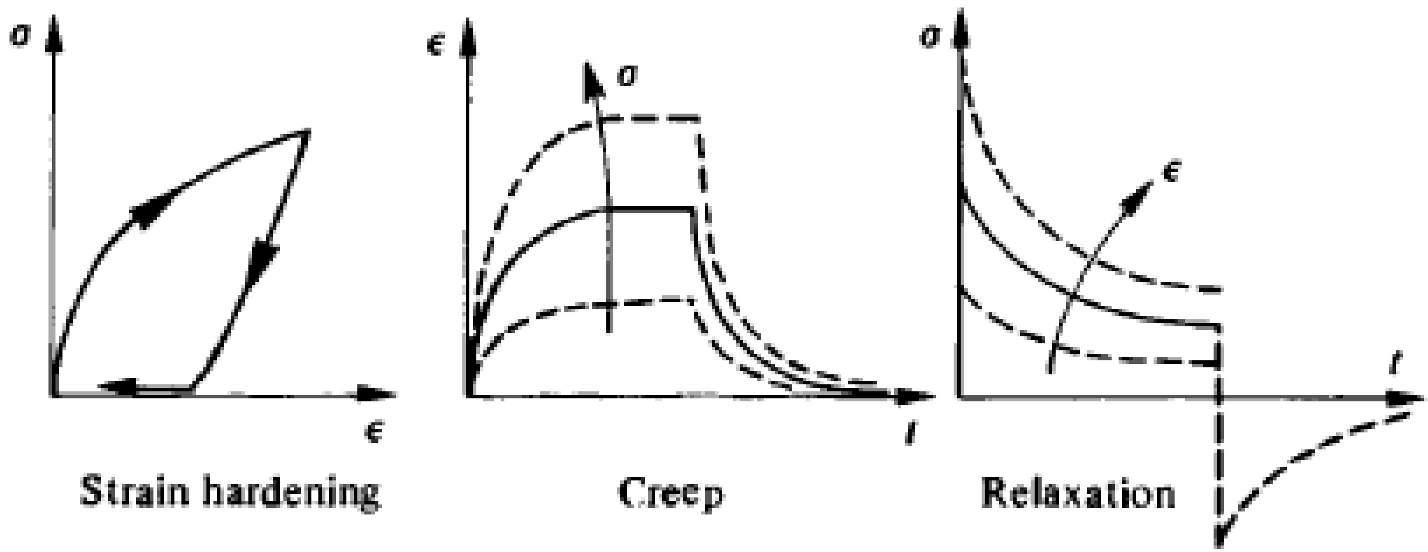


$$\sum A_i \overset{(i)}{\sigma} = \sum a_i \overset{(i)}{\varepsilon}$$

$$E_{rel}(t) = E_0 + \sum E_i\, e^{-\frac{E_i}{\eta_i}t}$$

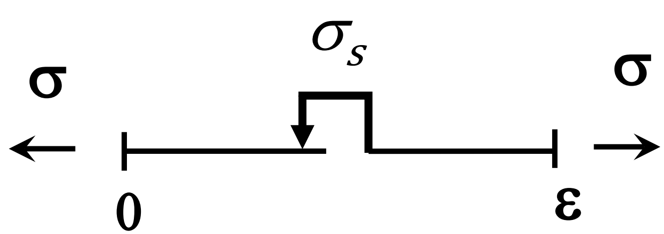
Material models

- Viscoelastic response



Material models

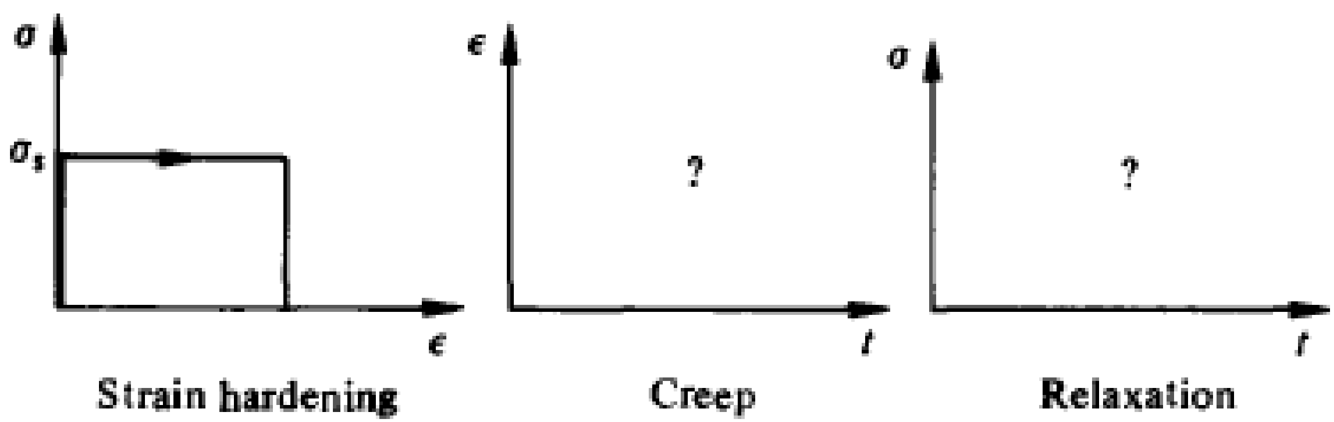
- Rigid perfectly plastic solid



$$|\sigma| < \sigma_s \Rightarrow \quad \varepsilon=0$$

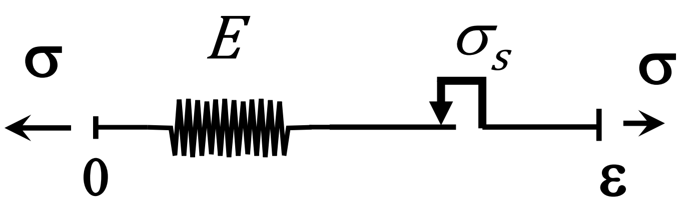
$$\sigma = \sigma_s \mathrm{Sgn}\,(\dot{\varepsilon}) \Rightarrow \varepsilon = \varepsilon_p$$

(Permanent strain negligible until a stress threshold and then arbitrary)



Material models

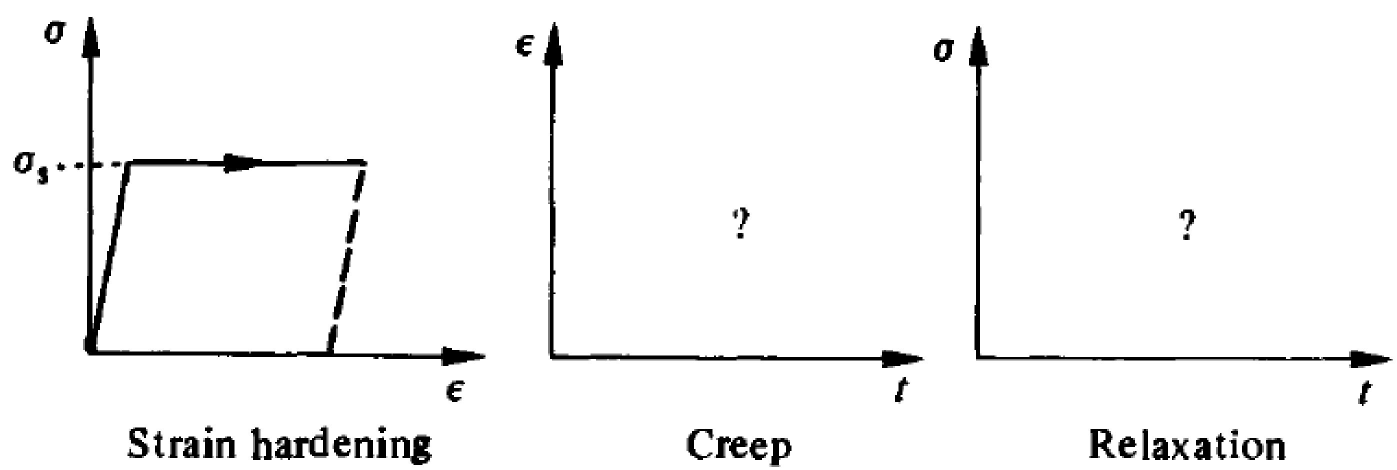
- Elastic perfectly plastic solid



$$|\sigma| < \sigma_s \Rightarrow \quad \varepsilon = \varepsilon_e = \frac{\sigma}{E}$$

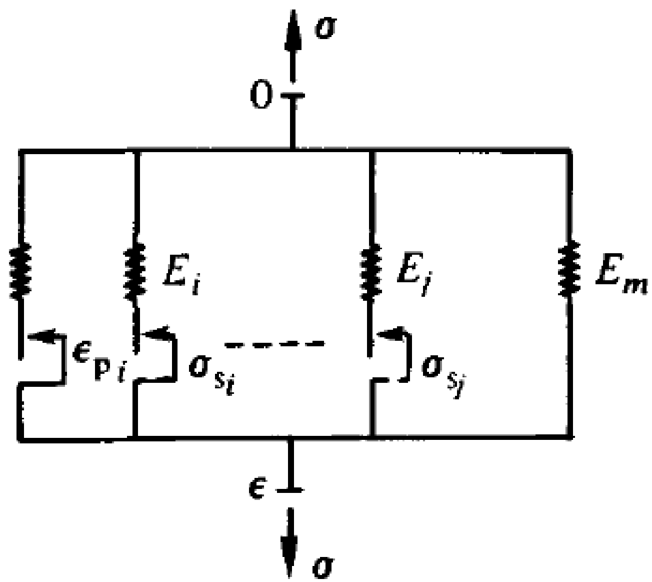
$$\sigma = \sigma_s \mathrm{Sgn}\,(\dot{\varepsilon}) \Rightarrow \varepsilon = \varepsilon_e + \varepsilon_p$$

(Permanent strain negligible until a stress threshold and then arbitrary)



Material models

- Elastoplastic hardening solid



$$|\sigma| < \sigma_s \Rightarrow \quad \varepsilon = \varepsilon_e = \frac{\sigma}{E}$$

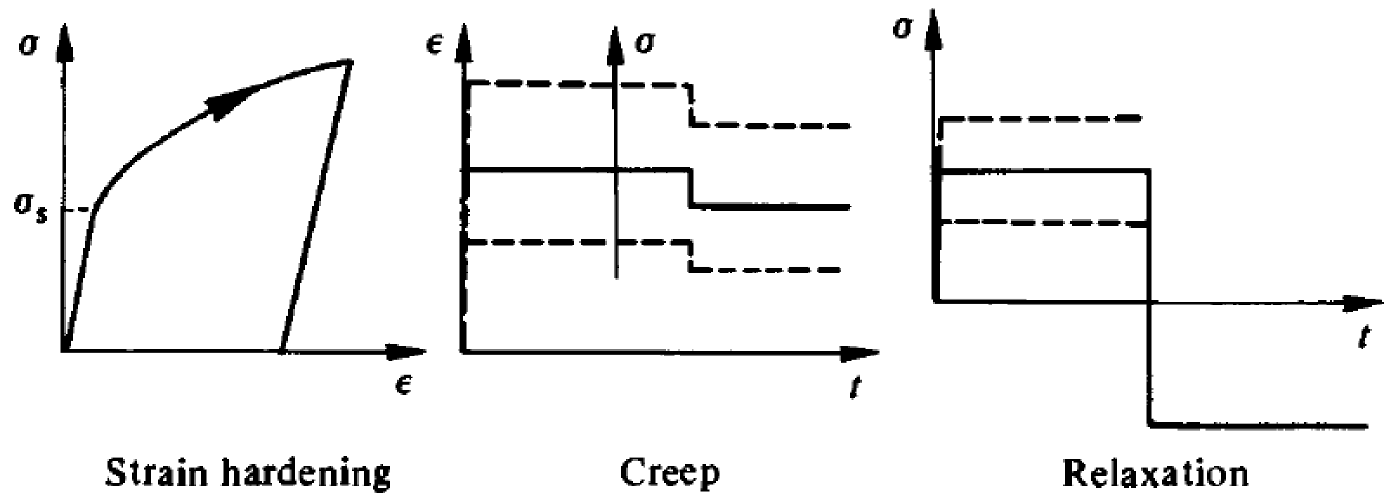
$$|\sigma| \geq \sigma_s \Rightarrow \varepsilon = \varepsilon_e + \varepsilon_p = \frac{\sigma}{E} + g(\sigma)$$

$$\sigma = \sum_{1}^{j} \sigma_{s_i} + \sum_{j+1}^{m} E_i \epsilon$$

$$\epsilon = \begin{cases} \dfrac{\sigma_{s_i}}{E_i} + \epsilon_{p_i} & \sigma_i = \sigma_{s_i} \\ \dfrac{\sigma_i}{E_i} & \sigma_i < \sigma_{s_i} \end{cases}$$

Material models

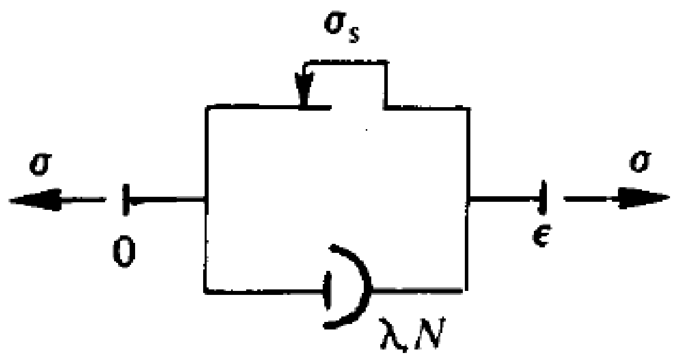
- Elastoplastic hardening solid



Time independent material with single-valued deformation during loading. Elastic unloadings.

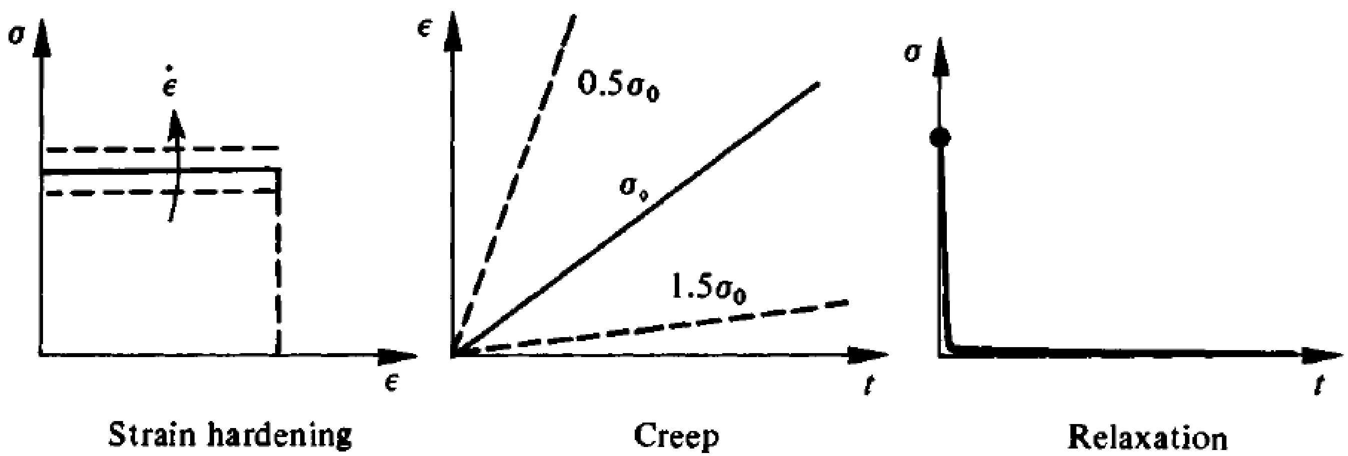
Material models

- Perfectly viscoplastic solid



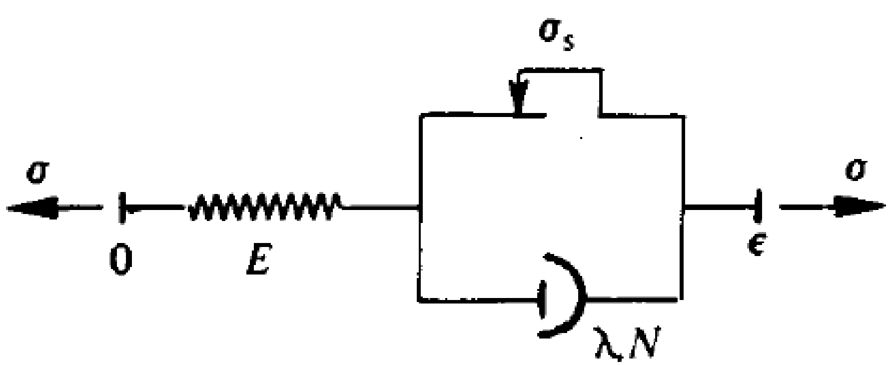
$$|\sigma| < \sigma_s \Rightarrow \quad \varepsilon = 0$$

$$\sigma = \sigma_s \mathrm{Sgn}\ (\dot{\varepsilon}) \Rightarrow \dot{\varepsilon}_p = \left(\frac{\sigma}{\lambda}\right)^N$$



Material models

- Elastic perfectly viscoplastic solid

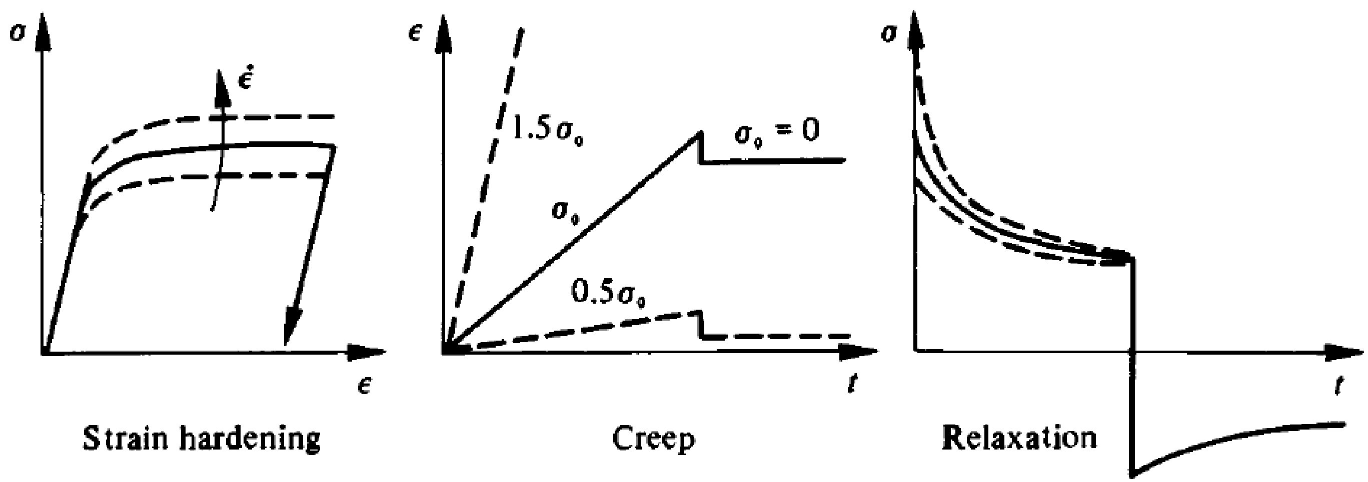


Bingham-Norton

$$|\sigma| < \sigma_s \Rightarrow \quad \varepsilon = \varepsilon_e = \frac{\sigma}{E}$$

$$|\sigma| \geq \sigma_s \Rightarrow \varepsilon = \varepsilon_e + \varepsilon_p$$

$$\sigma = E\,\varepsilon_e = \sigma_s + \lambda \dot{\varepsilon}^{\frac{1}{N}}$$



Strain hardening   Creep   Relaxation

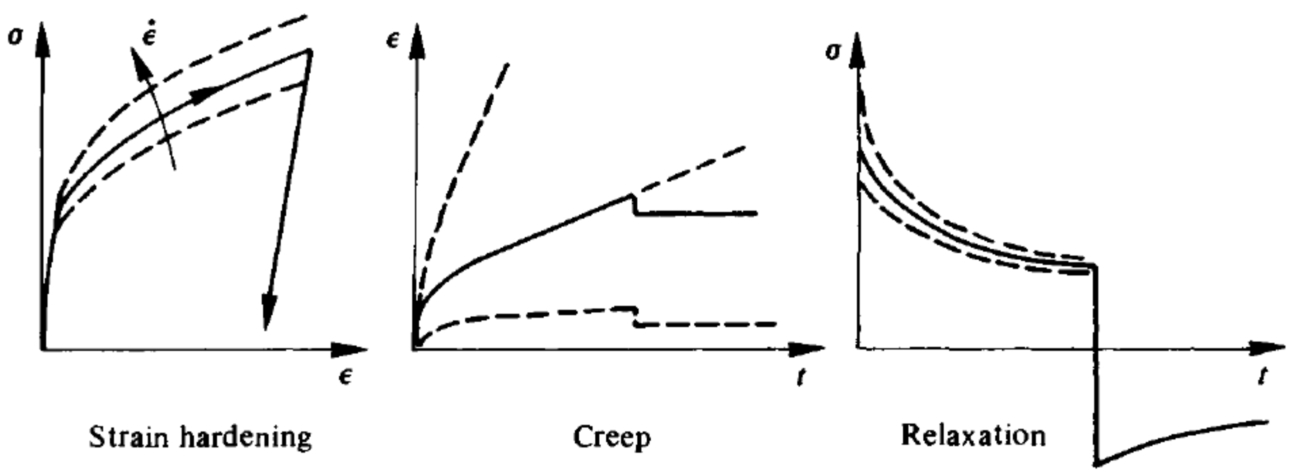
Material models

- Elastoviscoplastic hardening solid

$$|\sigma| < \sigma_s \Rightarrow \quad \varepsilon = \varepsilon_e = \frac{\sigma}{E}$$

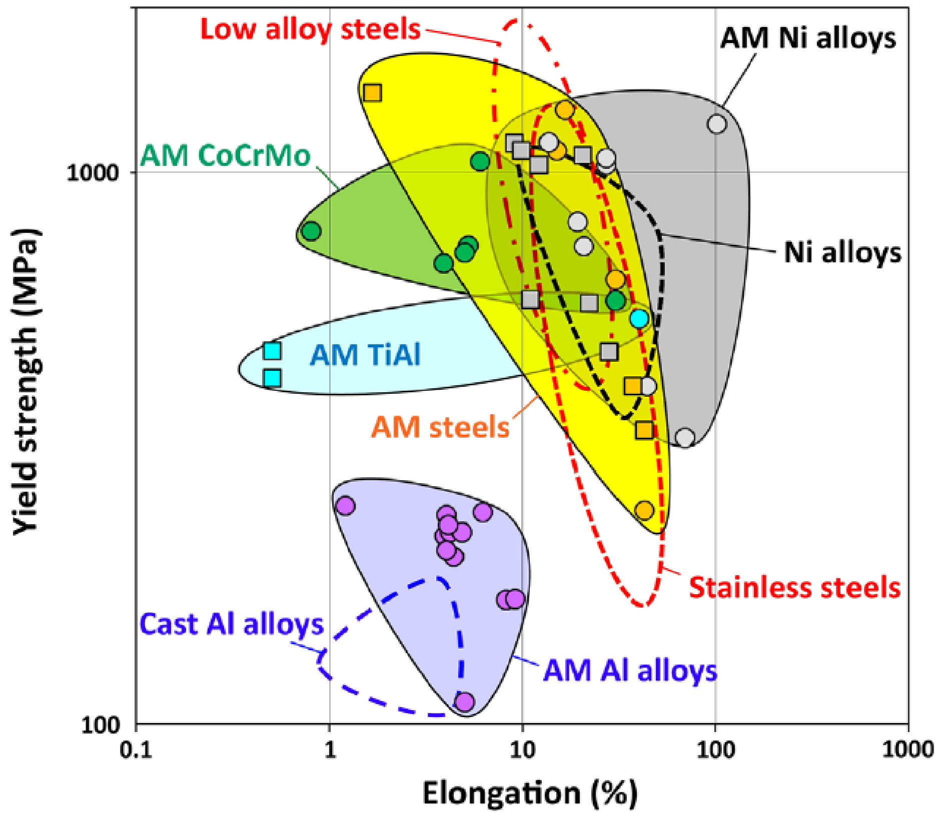
$$|\sigma| \geq \sigma_s \Rightarrow \varepsilon = \varepsilon_e + \varepsilon_p \qquad \sigma = E\,\varepsilon_e = f\left(\varepsilon_p, \dot{\varepsilon}_p, T\right)$$

$$\sigma = \left(A + B\varepsilon_p^n\right)\left(1 + C\ln\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)\left(1 - \left(\frac{T - T_0}{T_m - T_0}\right)^m\right)$$ Johnson-Cook



Examples of mechanical behaviour

- Metals



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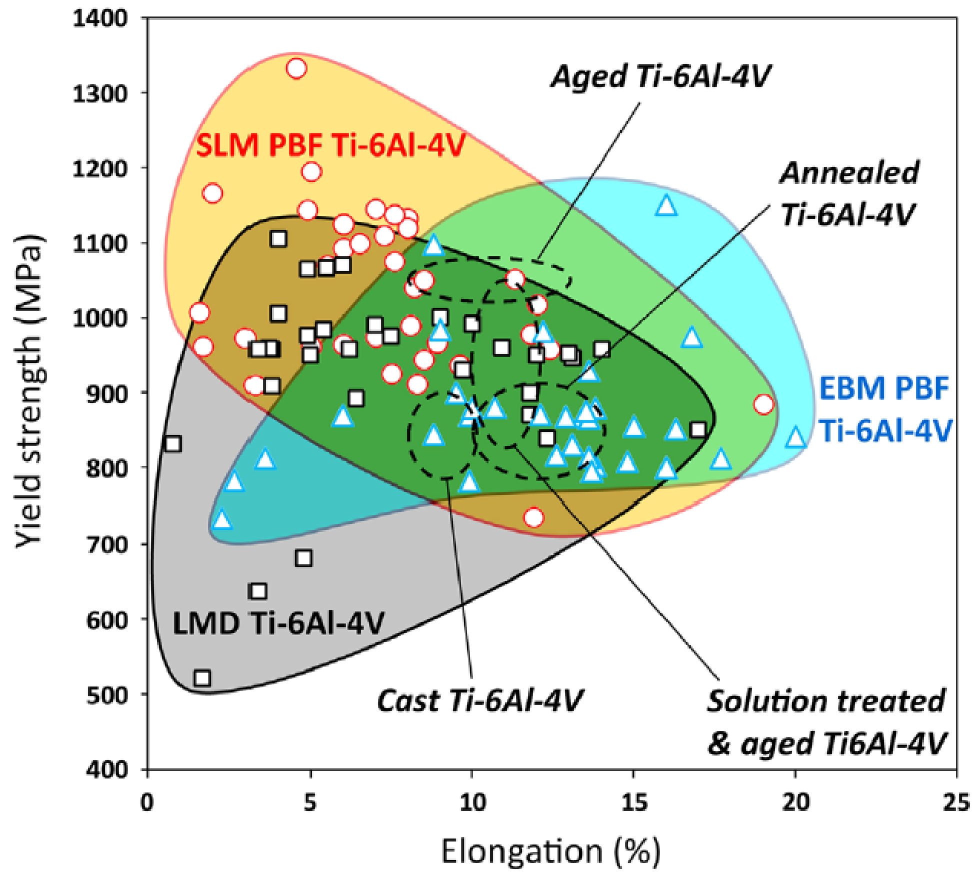
Examples of mechanical behaviour

- Ti-6Al-4V



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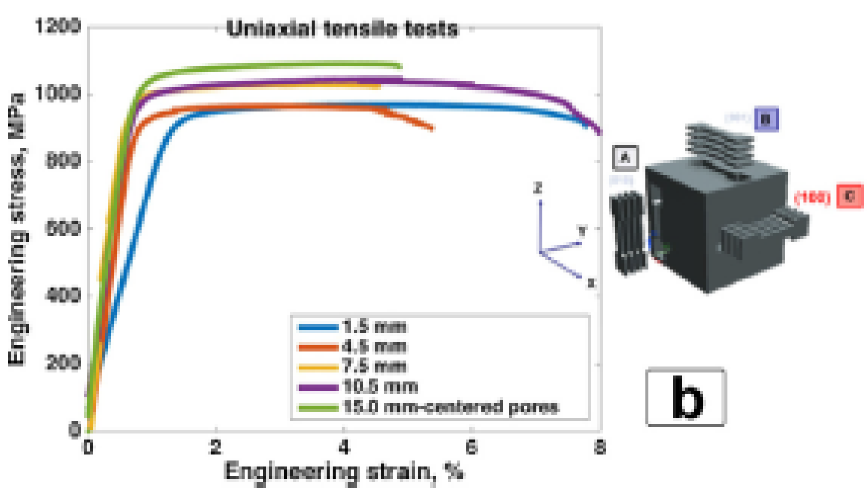
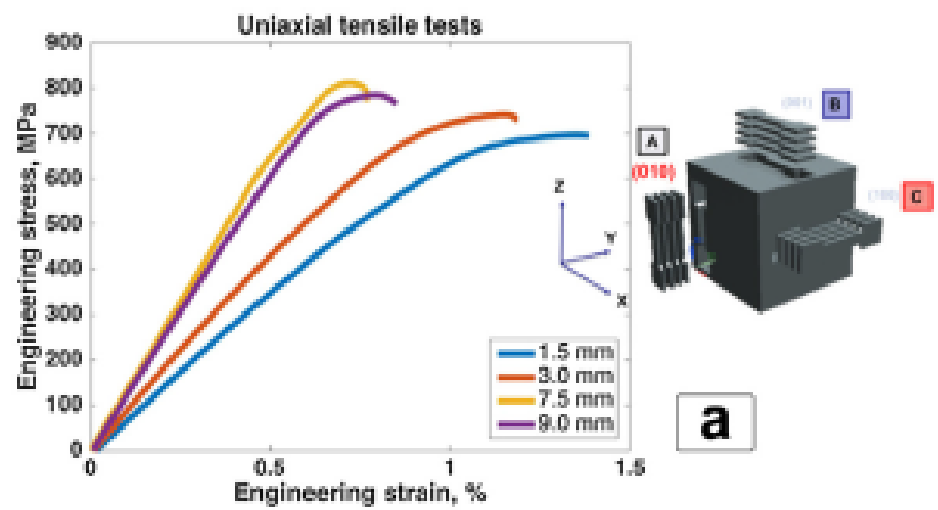
Examples of mechanical behaviour

- Ti-6Al-4V

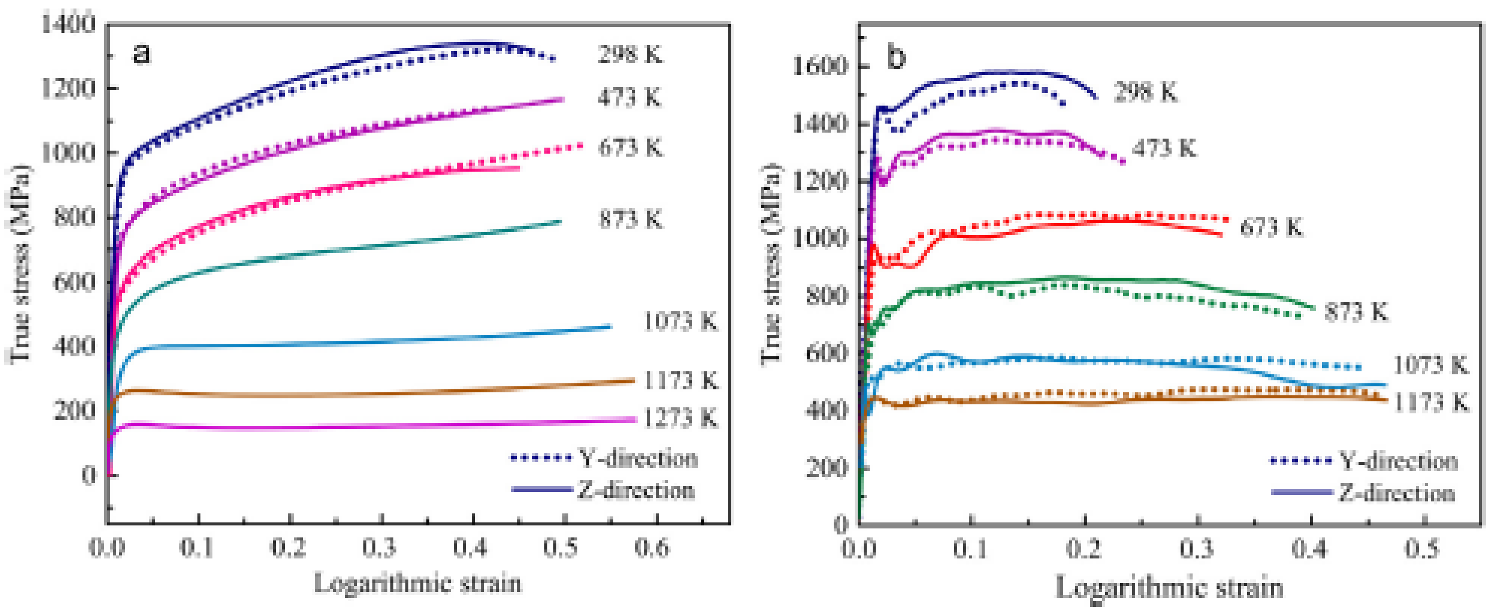




A. Azarniva et al. Journal of Alloys and Compounds 804 (2019) 163-191

Examples of mechanical behaviour

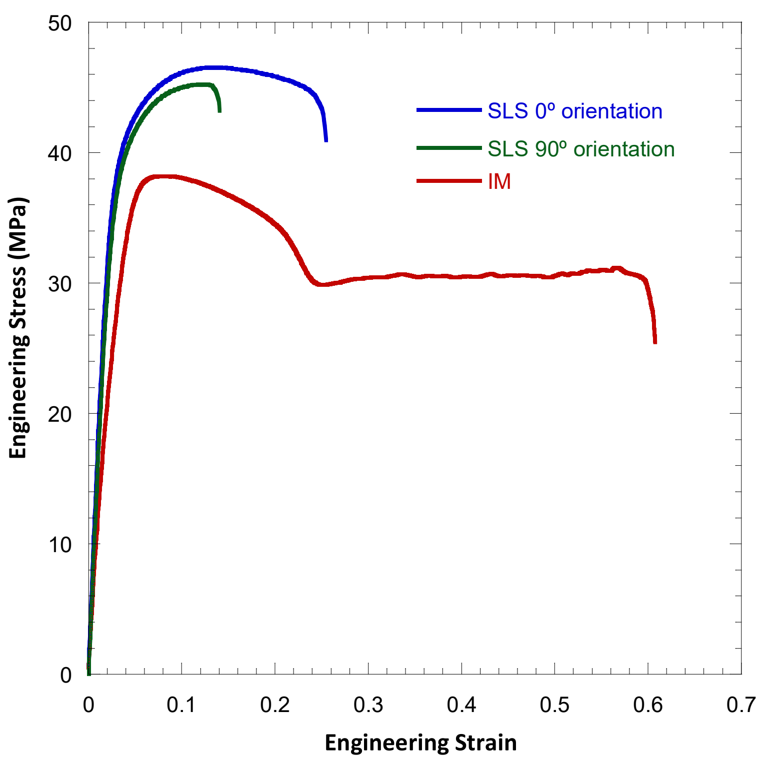
- LMD Ti-6Al-4V



P.-H. Li et al. / Materials Science & Engineering A 647 (2015) 34–42

Examples of mechanical behaviour

- Polymers: PA12



	E (GPa)	σ_T (MPa)	υ	ε_{max} (%)
SLS 0°	1.64 ± 0.03	47 ± 1	0.43 ± 0.02	27 ± 2
SLS 90°	1.58 ± 0.04	44 ± 2	0.41 ± 0.01	10 ± 4
IM	1.34 ± 0.04	41 ± 1	0.46 ± 0.02	64 ± 4

A. Cano. PhD Thesis. Universidad Rey Juan Carlos (2021).

Bibliography

- Mechanics of Solid Materials
  J. Lemaitre and J.L. Chaboche
  Cambridge University Press